Functions

Functions are another way of writing equations:

These mathematical statements all mean the same!

$y = 2x + 3$ – linear equation

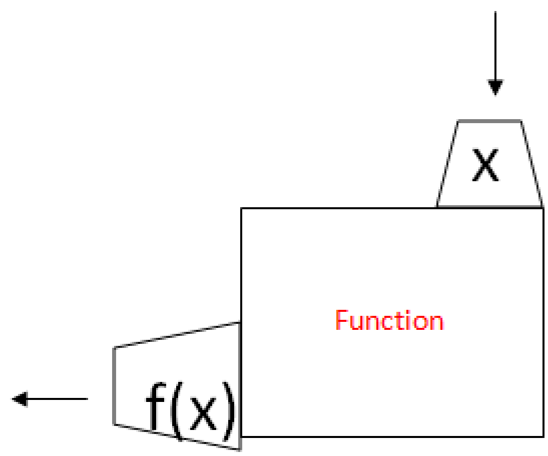
$$\left.\begin{aligned} f(x) &= 2x + 3 \\ g(x) &= 2x + 3 \\ h(a) &= 2a + 3 \end{aligned}\right\} \text{linear functions}$$

Notice y is replaced with f(x), g(x), even h(a).

This is function notation. They all mean exactly the same thing! You graph all of these exactly as you would y = 2x +3. We are just using a different notation! Functions do not have to be linear.

Evaluating Functions

Evaluating functions is no different from putting a number or expression "into a vending machine", we related functions to a vending machine. You "input" money and your "output" is candy or chips! We're going to go back to that visual as we evaluate functions. We are going to "input" a number and our "output" is the answer!



Page 2 has a number of examples:

Functions

$f(x) = 3x - 7$

$f(-5) = 3(-5) - 7$
$= -15 - 7$

$f(-5) = -22$

$f(4x) = 3(4x) - 7$

$f(4x) = 12x - 7$

$f(8x + 11) = 3(8x + 11) - 7$
$= 24x + 33 - 7$

$f(8x + 11) = 24x - 6$

$g(x) = x^2$

$g(-5) = (-5)^2$

$g(-5) = 25$

$g(4x) = (4x)^2$
$= (4x)(4x)$

$g(4x) = 16x^2$

$g(8x + 11) = (8x + 11)^2$
$= (8x+11)(8x+11)$
$= 64x^2 + 88x + 88x + 121$

$g(8x + 11) = 64x^2 + 176x + 121$

Use f(x) and g(x) from above and find:

1) $f(9) =$

2) $g(9) =$

3) $f(7x) =$

4) $g(7x) =$

5) $f(5x - 7) =$

6) $g(5x - 7)$

Functions

Composition of Functions

Compositions of Functions

When the input in a function is another function, the result is called a ***composite function.*** If

$f(x) = 3x + 2$ and $g(x) = 4x - 5$

then $f[g(x)]$ is a composite function. The statement $f[g(x)]$ is read " f of g of x" or "the composition of f with g." $f[g(x)]$ can also be written as

$$(f \circ g)(x) \quad \text{or} \quad f \circ g(x)$$

The symbol between f and g is a small open circle. When choosing the order of which function to use first you work "inside out". For f[g(x)] you would evaluate the g(x) (inside) first and then substitute your result into f(x). When replacing one function with another, be very careful to get the order correct because compositions of functions are not necessarily commutative (as you'll see).

$f(x) = 7x + 2$

$g(x) = x^2 + 8$

f[g(-2)] =

Find g(-2) first:
$g(-2) = (-2)^2 + 8$
$= 4 + 8$
$= 12$

Now plug 12 into f(x):
$f(12) = 7(12) + 2$
$= 84 + 2$
$= 86$

f[(g(-2)] = 86

g[f(-2)] =

Find f(-2) first:
$f(-2) = 7(-2) + 2$
$= -14 + 2$
$= -12$

Now plug -12 into g(x)
$g(-12) = (-12)^2 + 8$
$= 144 + 8$
$= 152$

g[f(-2)] = 152

Functions

$f(x) = 7x + 2$ $g(x) = x^2 + 8$

$f[g(x)] =$

Find g(x) first:
$g(x) = x^2 + 8$

Now plug $x^2 + 8$ into f(x):
$f(x^2 + 8) = 7(x^2 + 8) + 2$
$= 7x^2 + 56 + 2$
$= 7x^2 + 58$

$f[g(-2)] = 7x^2 + 58$

$g[f(x)] =$

Find f(x) first:
$f(x) = 7x + 2$

Now plug $7x + 2$ into g(x)
$g(7x+2) = (7x + 2)^2 + 8$
$= (7x+2)(7x+2) + 8$
$= 49x^2 + 14x+14x+ 4 + 8$
$= 49x^2 + 28x + 4 + 8$
$= 49x^2 + 28x + 12$

$g[f(x)] = 49x^2 + 28x + 12$

$f(x) = 7x + 2$ $g(x) = x^2 + 8$

$f[g(5x+2)] =$

$g(5x+2) = (5x + 2)^2 + 8$
$(5x+2)(5x+2) + 8$
$25x^2 +10x+10x + 4 + 8$
$25x^2 + 20x + 4 + 8$
$25x^2 + 20x + 12$

$f(25x^2+20x+12) = 7(25x^2+20x+12) + 2$
$175x^2 + 140x + 84 + 2$
$175x^2 + 140x + 86$

$f[g(5x+2)] = 175x^2 + 140x + 86$

$g(f(5x + 2)] =$

$f(5x + 2) = 7(5x + 2) + 8$
$35x + 14 + 8$
$35x + 22$

$g(35x+22) = (35x+22)^2 + 8$
$(35x+22)(35x+22) + 8$
$1{,}225x^2 +770x+770x+ 484 + 8$
$1{,}225x^2 + 1{,}540x + 484 + 8$
$1{,}225x^2 + 1{,}540x + 492$

$g[f(5x+2)] = 1{,}225x^2 + 1{,}540x + 492$

Functions

Functions: Answer 1-20 using f(x) and g(x)

$$f(x) = 3x + 5 \qquad\qquad g(x) = x^2$$

1) f(4) =

2) g(4) =

3) f(-6) =

4) g(-6) =

5) f(2x) =

6) g(2x) =

7) f(x+2) =

8) g(x+2) =

Functions

$f(x) = 3x + 5$ $g(x) = x^2$

9) $f(g(7))=$

10) $g(f(7))=$

11) $f(g(-2))=$

12) $g(f(-2))=$

Functions

$f(x) = 3x + 5 \qquad g(x) = x^2$

13) f(g(x))=

14) g(f(x)) =

15) f(g(x +5))=

16) g(f(x+5)) =

Functions

$f(x) = 3x + 5$ $\qquad$ $g(x) = x^2$

17) f(g(2x +1)) =

18) g(f(2x +1))=

19) f(g(3x-2))=

20) g(f(3x-2))=

Functions

$f(x) = 4x - 7$ $\qquad$ $g(x) = x^2 - 5$

1) $f(7) =$

2) $g(7) =$

3) $f(-5) =$

4) $g(-5) =$

5) $f(4x) =$

6) $g(4x) =$

7) $f(3x - 1) =$

8) $g(3x - 1) =$

Functions

$f(x) = 4x - 7$ $\qquad$ $g(x) = x^2 - 5$

9) f(g(8))=

10) g(f(8))=

11) f(g(-3))=

12) g(f(-3))=

Functions

$f(x) = 4x - 7$ $g(x) = x^2 - 5$

13) f(g(x))=

14) g(f(x)) =

15) f(g(x - 11))=

16) g(f(x - 11)) =

Functions

$f(x) = 4x - 7$ $\qquad$ $g(x) = x^2 - 5$

17) f(g(4x +7)) =

18) g(f(4x +7))=

19) f(g(5x-2))=

20) g(f(5x-2))=

Functions

Answer Key (Pages 5-8):

1) 17	**2) 16**	**3) -13**	**4) 36**
5) $6x + 5$	**6) $4x^2$**	**7) $3x+11$**	**8) $x^2 + 4x + 4$**
9) 152	**10) 676**	**11) 17**	**12) 1**
13) $3x^2 + 5$	**14) $9x^2 + 30x + 25$**	**15) $3x^2 + 30x + 30$**	**16) $9x^2 + 120x +$**
40017) $8x^2 + 8x + 7$	**18) $36x^2 + 96x + 64$**	**19) $27x^2 – 18x + 17$**	**20) $81x^2 – 18x + 1$**

Answer Key (pages 9-12)

1) 21	**2) 44**	**3) -27**	**4) 20**
5) $16x- 7$	**6) $16x^2 – 5$**	**7) $12x – 11$**	**8) $9x^2 – 6x – 4$**
9) 229	**10) 620**	**11) 9**	**12) 356**
13) $4x^2 – 27$	**14) $16x^2 – 56x + 44$**	**15) $4x^2 – 88x + 457$**	**16) $16x^2 – 408x + 2{,}601$**
17) $64x^2 + 112x + 169$	**18) $256x^2 + 672x + 436$**	**19) $100x^2 – 80x – 11$**	**20) $400x^2 – 600x + 220$**